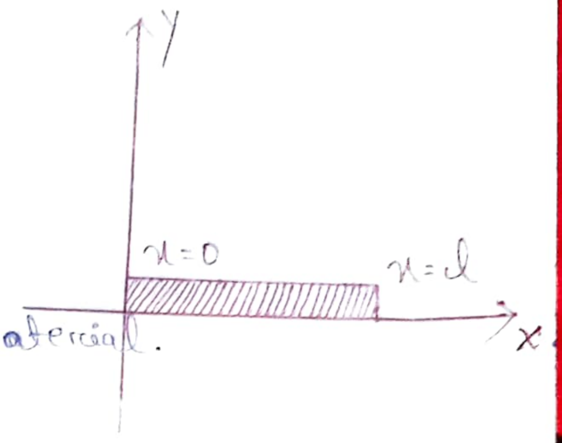
HEAT EQUATION :-

Derevation :-



$$\frac{\delta u}{\delta t} = c^2 \frac{\delta^2 u}{\delta x^2} \quad \text{———} \; (1)$$

where, c^2 = Constant.

diffusivity of the material.

U = variable.

$U(0, t) = U(l, t) = 0$

and, $U(x, 0) = f(x)$ at $t = 0$

It is solve by separation of variable method.

Let the Complete soln of

$$U(x, t) = X_{(x)} \cdot T_{(t)} \quad \text{———} \; (2)$$

Now,

$$T'X = c^2 X'' T.$$

$$\frac{T'}{c^2 T} = \frac{X''}{X} = k \; (\text{Say})$$

Now,

$$\frac{T'}{c^2 T} = k.$$

$$\Rightarrow \frac{T'}{T} = c^2 k \quad \text{———} \; (3)$$

and $\frac{X''}{X} = k.$

$$\Rightarrow X'' - kX = 0 \quad \text{———} \; (4)$$

There are 3 cases ariages for the constant value k.

Case-I :—

when $k = 0$

$T' = 0$

Integrating

$T = C_1$

$X'' = 0$

Integrating

$X' = C_2$

Again Integrating

$X = C_2 x + C_3$

Hence the complete sol^n of

$u(x, t) = (C_2 x + C_3) C_1$

$= C_1 (C_2 x + C_3)$ —— (5)

Case-II :—

when k is +ve value.

when $k > 0$

Let $k = p^2$ (p is any integer number)

From eq^n (3) & (4)

$\frac{T'}{T} = c^2 p^2$

Integrating

$\log T = c^2 p^2 t + \log c_1$

$T = e^{c^2 p^2 t} c_1$

$= c_1 e^{c^2 p^2 t}$

$\frac{X''}{X} = p^2$

$X'' = p^2 X$

$X'' - p^2 X = 0$

The Auxiliary Eqn

$$m^2 - p^2 = 0$$

$$m = \pm p$$

The Soln is

$$X = c_2 e^{px} + c_3 e^{-px}$$

Hence the complete soln of eqn (1)

$$u(x,t) = \left(c_2 e^{px} + c_3 e^{-px}\right) c_1 e^{c^2 p^2 t} \qquad \text{——— (6)}$$

which is the complete soln.

Case - III :—

when k is -ve value.

$$k < 0$$

Let $k = -p^2$ (p is any number)

Now from eqn (3) & (4)

$$\frac{T'}{T} = -p^2 c^2$$

Integrating

$$\log T = -p^2 c^2 t + \log c_1$$

$$T = c_1 e^{-p^2 c^2 t}.$$

$$\frac{X''}{X} = -p^2$$

$$X'' = -p^2 X$$

$$X'' + p^2 X = 0$$

The Auxiliary Eqn is

$$m^2 + p^2 = 0$$

$$m = \pm ip$$

The Soln is

$$X = c_2 \cos px + c_3 \sin px.$$

Hence the complete soln of (1) is

$$u(x,t) = \left(c_2 \cos px + c_3 \sin px\right) - p^2 c^2 + c_1 e \qquad \text{——— (7)}$$

Since the physical nature of any heat flow eq^n are show by eq^n (7) which become complete sol^n of heat eq^n (1)

Applying the condition.

put $x=0$ in Eq^n (7)

$$U(0,t) = c_1 e^{-p^2c^2t}(c_2 \cdot 1 + 0)$$

$$= c_1 e^{-p^2c^2t}(c_2)$$

$$c_2 c_1 e^{-p^2c^2t} = 0$$

$$\Rightarrow c_2 = 0.$$

Hence the complete sol^n is

$$U(x,t) = c_1 e^{-p^2c^2t}(c_3 \sin px)$$

$$= A e^{-p^2c^2t} \sin px.$$

where, $A = c_1c_3$ —— (8)

Again,

put $x = l$ in Eq^n (8)

$$U(l,t) = Ae^{-p^2c^2t} \sin pl.$$

$$Ae^{-p^2c^2t} \sin pl = 0$$

$$\sin pl = 0$$

$$pl = n\pi$$

$$p = \frac{n\pi}{l}.$$

Hence the complete sol^n of (1) is

$$u(x,t) = Ae^{-\frac{n^2\pi^2}{l^2}c^2t} \cdot \sin\left(\frac{n\pi x}{l}\right) \quad \text{—— (9)}$$

The general sol^n of (1) is

$$u(x,t) = \sum_{n=1}^{\infty} A_n e^{-\frac{n^2\pi^2c^2t}{l^2}} \sin\left(\frac{n\pi x}{l}\right) \quad \text{—— (10)}$$

Now put $t=0$ in eqⁿ (10)

$$U(x,0) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{l}\right) = f(x)$$

or

$$f(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{l}\right)$$

Now multiplying both side by $\sin\left(\frac{m\pi x}{l}\right)$ and then integrating w.r.t. 'x' between the limit $x=0$ to $x=l$.

$$\int_0^l f(x)\sin\left(\frac{m\pi x}{l}\right)dx = \sum_{n=1}^{\infty} A_n \int_0^l \sin\left(\frac{n\pi x}{l}\right)\sin\left(\frac{m\pi x}{l}\right)dx$$

$$\Rightarrow \int_0^l f(x)\sin\left(\frac{m\pi x}{l}\right)dx = A_m \cdot l/2$$

$$\int_0^l \sin\left(\frac{n\pi x}{l}\right)\sin\left(\frac{m\pi x}{l}\right)dx = \begin{cases} l/2 , & m=n \\ 0 , & m \neq n \end{cases}$$

$$\Rightarrow A_m = 2/l \int_0^l f(x)\sin\left(\frac{m\pi x}{l}\right)dx .$$

Hence the final general solⁿ of Heat flow eqⁿ (1) is

$$U(x,t) = \sum_{n=1}^{\infty} A_n e^{-\frac{n^2\pi^2c^2t}{l^2}} \sin\left(\frac{n\pi x}{l}\right)$$

where,

$$A_n = 2/l \int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx .$$

WAVE EQUATION :—

Derevation :—

$$\frac{\delta^2 u}{\delta t^2} = c^2 \frac{\delta u}{\delta x^2}$$

where $c^2 = T/m$ = Constant.

Subject to codition.

$$U(0,t) = U(l,t) = 0$$

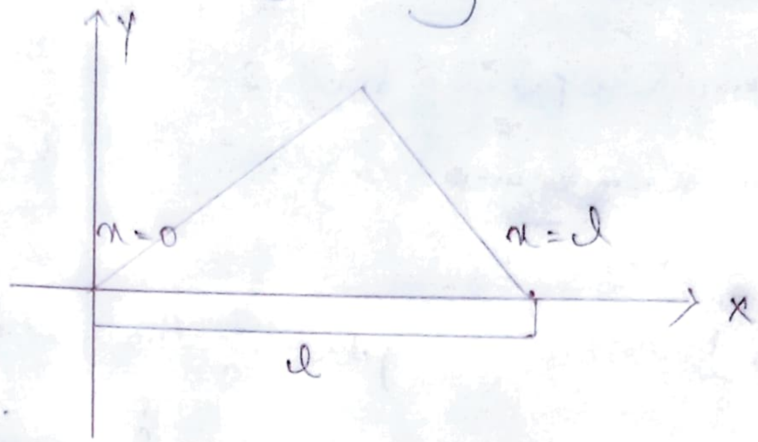
and initial condition.

$$U(x,0) = f(x)$$

and $\frac{\delta u}{\delta t} = 0$ at $t = 0$

where,

l = length of string or curve.



This wave eqn solve by separation of variable method.

Soln Given,

$$\frac{\delta^2 u}{\delta t^2} = c^2 \cdot \frac{\delta^2 u}{\delta x^2} \quad \text{--- (1)}$$

Let the complete soln of wave eqn is

$$u(x,t) = X_{(x)} \cdot T_{(t)} \quad \text{--- (2)}$$

$$T''X = c^2 X'' T$$

$$\Rightarrow \frac{T''}{c^2 T} = \frac{X''}{X} = k \text{ (say)}$$

$$\Rightarrow \frac{T''}{c^2 T} = k.$$

$$\Rightarrow T'' = c^2 + k.$$

$$\Rightarrow T'' - c^2 + k = 0 \quad \text{--- (3)}$$

$$\frac{X''}{X} = k.$$

$$\Rightarrow X'' - Xk = 0 \quad \text{--- (4)}$$

Here there are 3 cases arises for the value of k.

Case-I :—

when, $k=0$ Then we have.

$T''=0$

$T'=c_1$

$T=c_1t+c_2$.

and, $X''=0$

Integrated.

$X'=c_3$

Again Integrating

$X=c_3x+c_4$

Hence the complete solⁿ is

$u(x,t)=(c_1t+c_2)(c_3x+c_4)$ — (5)

Case-II :—

when k is positive, $k>0$.

$k=p^2$ (where p is any number)

So we have eqⁿ (3) and (4) gives.

$T''-p^2c^2T=0$

$(D^2-p^2c^2)T=0$

The Auxiliary Eqⁿ is

$m^2-p^2c^2=0$

$\Rightarrow m^2=p^2c^2$

$\Rightarrow m=\sqrt{p^2c^2}$

$\Rightarrow m=\pm pc$

$T=c_1e^{pct}+c_2e^{-pct}$.

and, $X''-p^2X=0$

$(D^2-p^2)X=0$

$\Rightarrow$ The Auxiliary eqⁿ is

$m^2-p^2=0$

$\Rightarrow m^2=p^2$

$\Rightarrow m=\pm p$

$X=c_3e^{px}+c_4e^{-px}$.

Hence the Complete Solⁿ is

$$u(x,t) = \left(c_1 e^{pct} + c_2 e^{-pct}\right)\left(c_3 e^{px} + c_4 e^{-px}\right) \quad \text{--- } (6)$$

Case - III :-

when k is negative $k < 0$

$k = -p^2$ (where p is any number)

Now from Eqⁿ (3) & (4)

$$T'' + p^2c^2T = 0$$

$$(D^2 + p^2c^2)T = 0$$

The Auxiliary Eqⁿ is

$$m^2 + p^2c^2 = 0$$

$$\Rightarrow m^2 = -p^2c^2$$

$$\Rightarrow m = \sqrt{-p^2c^2} = \pm ipc$$

$$T = c_1 \cos pct + c_2 \sin pct.$$

$$X'' + p^2X = 0$$

$$(D^2 + p^2)X = 0$$

The Auxiliary Eqⁿ is.

$$m^2 + p^2 = 0$$

$$\Rightarrow m^2 = -p^2$$

$$\Rightarrow m = \sqrt{-p^2}$$

$$\Rightarrow m = \pm ip$$

$$X = c_3 \cos px + c_4 \sin px.$$

Hence the Complete Solⁿ is

$$u(x,t) = \left(c_1 \cos pct + c_2 \sin pct\right)\left(c_3 \cos px + c_4 \sin px\right) \quad \text{--- } (7)$$

Since the periodic function are present in eq^n (7) and It shows this physical nature of wave 7 is the only sol^n of wave eq^n which become complete Solution eq^n (1)

By applying the Boundary Condition put $x=0$ in eq^n (7)

$$u(0,t) = c_3(c_1\cos pct + c_2\sin pct)$$

$$\Rightarrow c_3 = 0$$

Now,

Hence the complete sol^n of Eq^n (1) is

$$u(x,t) = c_4 \sin px\,(c_1\cos pct + c_2\sin pct) \quad \text{--- (8)}$$

Now again,

put $x=l$ in Eq^n (8)

$$u(l,t) = c_4 \sin pl\,(c_1\cos pct + c_2\sin pct)$$

$$c_4 \sin pl = 0$$

$$\sin pl = 0$$

$$pl = n\pi$$

$$\Rightarrow p = \frac{n\pi}{l}$$

Hence the complete sol^n of wave eq^n

$$u(x,t) = c_4 \sin\left(\frac{n\pi x}{l}\right)\left(c_1\cos\left(\frac{n\pi ct}{l}\right) + c_2\sin\left(\frac{n\pi ct}{l}\right)\right) \quad \text{--- (9)}$$

$$= c_1c_4\sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi ct}{l}\right) + c_2c_4\sin\left(\frac{n\pi x}{l}\right)\sin\left(\frac{n\pi ct}{e}\right)$$

$$= \sin\left(\frac{n\pi x}{l}\right)\left[A\cos\left(\frac{n\pi ct}{l}\right) + B\sin\left(\frac{n\pi ct}{l}\right)\right] \quad \text{--- (10)}$$

where,

$$A = c_1c_4$$

$$B = c_2c_4$$

By using the initial Condition.

$$\frac{\delta u}{\delta t} = 0 \text{ at } t = 0.$$

$$\Rightarrow \frac{\delta u}{\delta t} = \sin\left(\frac{n\pi x}{l}\right)\left[-A\sin\left(\frac{n\pi ct}{l}\right)\left(\frac{n\pi c}{l}\right) + B\cos\left(\frac{n\pi ct}{l}\right)\left(\frac{n\pi c}{l}\right)\right]$$

$$\Rightarrow \left.\frac{\delta u}{\delta t}\right]_{t=0} = \sin\left(\frac{n\pi x}{l}\right)\left[B\left(\frac{n\pi c}{l}\right)\right]$$

$B = 0$

Hence the complete solⁿ is

$$u(x,t) = A\sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi ct}{l}\right)$$

$n = 1, 2, 3 \ldots$

Hence the general solⁿ of wave eqⁿ

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi ct}{l}\right) \quad \text{—— (ii)}$$

To find the values of A_n put $t=0$ in eqⁿ (ii)

$$u(x,0) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{l}\right) = f(x)$$

or $$f(x) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{l}\right)$$

multiplying both side by $\sin\left(\frac{m\pi x}{l}\right)$ and then integrating w.r.t 'x' between the limit $x=0$ to $x=l$

$$\int_0^l f(x)\sin\left(\frac{m\pi x}{l}\right)dx$$

$$= \sum_{n=1}^{\infty} A_n \int_0^l \sin\left(\frac{n\pi x}{l}\right)\sin\left(\frac{m\pi x}{l}\right)dx.$$

$$\int_0^l \sin\left(\frac{n\pi x}{l}\right)\cdot\sin\left(\frac{m\pi x}{l}\right)dx = \begin{cases} l/2, & m=n \\ 0, & m\neq n \end{cases}$$

$$\int_0^l f(x)\sin\left(\frac{m\pi x}{l}\right)dx = A_m \cdot l/2.$$

$$A_m = 2/l \int_0^l f(x)\sin\left(\frac{m\pi x}{l}\right)dx.$$

or $$A_n = 2/l \int_0^l f(x)\cdot\sin\left(\frac{n\pi x}{l}\right)dx.$$

Now finally the general solution of one dimension wave eqn is

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi x}{l}\right) \cos\left(\frac{n\pi c t}{l}\right)$$

where $A_n = \int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx$

LAPLACE EQUATION :—

Consider a flow of Heat in a metal plate in the directions of its length (x-axis) and breadth (y-axis) respectively. where there is no flow of het along the direction of the normal (z-axis) to the plane of the rectangle.

The temperature at any point is independent of the z-co-ordinate and depends on x, y and t.

Consider the flow of heat in a rectangular plate with side δx and δy.

Let the metal be uniform thickness α, density ρ.

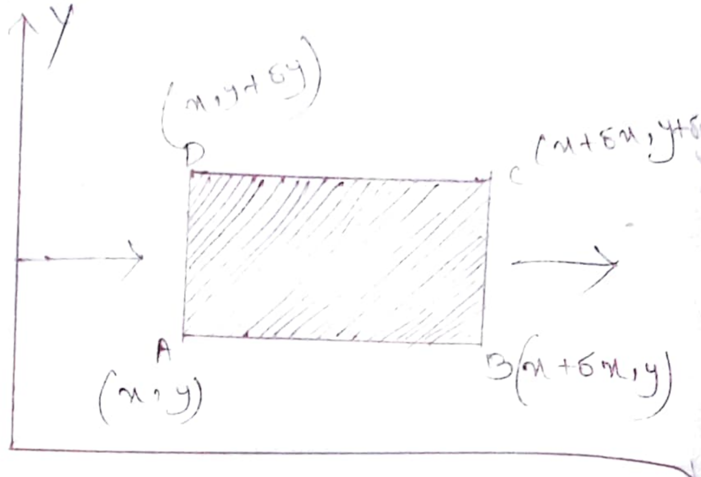
Specific heat σ and thermal conductivity k.

Quantity of heat enter the plate per second from the side AB

$$= -k\alpha\delta x\left(\frac{\delta u}{\delta y}\right)$$

Similarly, the quantity of heat that enters the plate per second from the side

$AB = -k\alpha\,\delta x\left(\frac{\delta u}{\delta x}\right)$



The quantity of heat flows out through the Sides CD and BC per seconds is $-k\alpha\,\delta x\left(\delta u/\delta y\right)$ and

$$-k\alpha\,\delta y\left(\frac{\delta u}{\delta x}\right)_{x+\delta x}.$$

Hence total gain of heat by the rectangular element ABCD per second.

$$= -k\alpha\,\delta x\left(\frac{\delta u}{\delta y}\right)_y - k\alpha\,\delta y\left(\frac{\delta u}{\delta x}\right)_x + k\alpha\,\delta x\left(\frac{\delta u}{\delta y}\right)_{y+\delta y}$$

$$+k\alpha\,\delta y\left(\frac{\delta u}{\delta x}\right)_{x+\delta x}$$

$$= -k\alpha(\delta x)(\delta y)\left[\frac{\left(\frac{\delta u}{\delta x}\right)_{x+\delta x} - \left(\frac{\delta u}{\delta x}\right)_x}{\delta x} + \frac{\left(\frac{\delta u}{\delta y}\right)_{y+\delta y} - \left(\frac{\delta u}{\delta y}\right)_y}{\delta y}\right]$$

The rate of gain of heat.

$$= \gamma f.\,\delta x\,\delta y\left(\frac{\delta u}{\delta t}\right)$$

$$k\alpha(\delta x)(\delta y)\left(\frac{\left(\frac{\delta u}{\delta x}\right)_{x+\delta x} - \left(\frac{\delta u}{\delta x}\right)_x}{\delta x} + \frac{\left(\frac{\delta u}{\delta y}\right)_{y+\delta y} - \left(\frac{\delta u}{\delta y}\right)_y}{\delta y}\right)$$

$$= \gamma f(\delta x)(\delta y)\,\frac{\delta u}{\delta t}.$$

Dividing both side $\alpha(\delta x)(\delta y)$

Taking limit $\delta x \to 0$
$\delta y \to 0$

$$k\left(\frac{\delta^2 u}{\delta x^2}+\frac{\delta^2 u}{\delta y^2}\right)=\left(\frac{\delta f}{\alpha}\right)\left(\frac{\delta u}{\delta t}\right)$$

$$\Rightarrow \frac{\delta u}{\delta t}=\left(\frac{k\alpha}{\gamma f}\right)\left(\frac{\delta^2 u}{\delta x^2}+\frac{\delta^2 u}{\delta y^2}\right)$$

$$\Rightarrow \frac{\delta u}{\delta t}=c^2\left(\frac{\delta^2 u}{\delta x^2}+\frac{\delta^2 u}{\delta y^2}\right)$$

where $c^2=\frac{k\alpha}{\gamma f}$

The distribution of temperature in the plate u does not change with t.

So $\frac{\delta u}{\delta t}=0$

$$\Rightarrow \frac{\delta^2 u}{\delta x^2}+\frac{\delta^2 u}{\delta y^2}=0$$

which is called Laplace Eqn in two dimension

Partial Differential eqn with variable co-efficient Eqn Reducible to canonical form:-

$$Rr+Ss+Tt+Pp+Qq+Zz=W \quad \text{---} \quad (1)$$

$$\eta=\eta(x,y)$$
$$\tau=\tau(x,y)$$

$$p=\frac{\delta z}{\delta x}=\frac{\delta z}{\delta \eta}\cdot\frac{\delta \eta}{\delta x}+\frac{\delta z}{\delta \tau}\cdot\frac{\delta \tau}{\delta x}$$

$$q=\frac{\delta z}{\delta y}=\frac{\delta z}{\delta y}\cdot\frac{\delta \eta}{\delta y}+\frac{\delta z}{\delta \tau}\cdot\frac{\delta \tau}{\delta y}$$

$$r=\frac{\delta^2 z}{\delta x^2}=\frac{\delta}{\delta x}\left(\frac{\delta z}{\delta x}\right)=\frac{\delta}{\delta x}p=\frac{\delta p}{\delta x}$$

$$=\frac{\delta^2 z}{\delta \eta^2}\left(\frac{\delta \eta}{\delta x}\right)^2+2\frac{\delta^2 z}{\delta \tau\delta \eta}\cdot\frac{\delta \eta}{\delta x}\cdot\frac{\delta z}{\delta x}+$$

$$\frac{\delta^2 z}{\delta \tau^2}\left(\frac{\delta \tau}{\delta x}\right)+\frac{\delta z}{\delta \eta}\cdot\frac{\delta^2 \eta}{\delta x^2}+\frac{\delta z}{\delta \tau}\cdot\frac{\delta^2 \tau}{\delta x^2}$$

$$s = \frac{\delta^2 z}{\delta y \delta x} = \frac{\delta p}{\delta y}$$

$$= \frac{\delta^2 z}{\delta \xi^2} \frac{\delta \xi}{\delta x} \frac{\delta \xi}{\delta y} + \frac{\delta^2 z}{\delta \eta \delta \xi}\left(\frac{\delta \xi}{\delta x} \frac{\delta \eta}{\delta y} + \frac{\delta \eta}{\delta x} \cdot \frac{\delta \xi}{\delta y}\right)$$

$$+ \frac{\delta^2 z}{\delta \eta^2} \frac{\delta \eta}{\delta x} \frac{\delta \eta}{\delta y} + \frac{\delta z}{\delta \xi} \frac{\delta^2 \xi}{\delta y \delta x} + \frac{\delta z}{\delta \eta} \cdot \frac{\delta^2 \eta}{\delta y \delta x}$$

$$t = \frac{\delta^2 z}{\delta y^2} = \frac{\delta q}{\delta y}$$

$$= \frac{\delta^2 z}{\delta \xi^2}\left(\frac{\delta \xi}{\delta y}\right)^2 + 2\frac{\delta^2 z}{\delta \xi \delta \eta} \cdot \frac{\delta \xi}{\delta y} \frac{\delta \eta}{\delta y} + \frac{\delta^2 z}{\delta \eta^2}\left(\frac{\delta \eta}{\delta y}\right)^2$$

$$+ \frac{\delta z}{\delta \xi} \frac{\delta^2 \xi}{\delta y^2} + \frac{\delta z}{\delta \eta} \frac{\delta^2 \eta}{\delta y^2}.$$

Exampl:—

$$xyr - (x^2-y^2)s - xyt + py - qy = 2(x^2-y^2) \quad \text{—} \quad (1)$$

Reduce the eqn in canonical form.

General formula:—

$$Rr + Ss + Tt + Pp + Qq + Zz = W$$

Soln $R\alpha^2 + S\alpha + T = 0$

1) $S^2 - 4RT > 0$ (Hyperbolic Eqn)

2) $S^2 - 4RT > 0$ (parabolic Eqn)

3) $S^2 - 4RT < 0$ (Elliptic Eqn)

Here,

$R = xy$

$S = -(x^2-y^2)$

Now put these eqn

$$S^2 - 4RT = \{-(x^2-y^2)\}^2 - 4 \cdot xy \cdot (-xy)$$

$$= (x^2-y^2)^2 + 4x^2y^2$$

$$= x^4 + y^4 - 2x^2y^2 + 4x^2y^2$$

$$= x^4 + y^4 - 2x^2y^2 = (x^2+y^2)^2 > 0$$

So It's gives a Hyperbolic eq^n.

Now, $R\alpha^2 + S\alpha + T = 0$

$\Rightarrow xy\alpha^2 + \{-(x^2-y^2)\}\alpha + (-xy) = 0$

$\Rightarrow xy\alpha^2 - (x^2-y^2)\alpha - xy = 0$

$\Rightarrow xy\alpha^2 - x^2\alpha + y^2\alpha - xy = 0$

$\Rightarrow x\alpha(y\alpha - x) + y(y\alpha - x) = 0$

$\Rightarrow (y\alpha - x)(x\alpha - y) = 0$

$\Rightarrow y\alpha - x = 0$ or $x\alpha + y = 0$

$\Rightarrow y\alpha = x$ or $x\alpha = -y$

$\Rightarrow \alpha_1 = x/y$ or $\alpha_2 = -y/x$.

Now, $\dfrac{dy}{dx} + \alpha_1(x,y) = 0$

$\dfrac{dy}{dx} + \alpha_2(x,y) = 0$

$\dfrac{dy}{dx} \; x/y = 0$

$\Rightarrow \dfrac{dy}{dx} = -x/y$

$\Rightarrow y\,dy = -x\,dx$.

Integrating

$\int y\,dy = -\int x\,dx$.

$\Rightarrow y^2/2 = -x^2/2 + \tau/2$

$\Rightarrow y^2/2 + x^2/2 = \tau/2$

$\Rightarrow y^2 + x^2 = \tau$

$\Rightarrow \tau = x^2 + y^2$

$\dfrac{dy}{dx} + (-y/x) = 0$

$\Rightarrow \dfrac{dy}{dx} - y/x = 0$

$\Rightarrow dy/dx = y/x$.

$$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$$

Integrating

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\Rightarrow \log y = \log x + \log \eta$$

$$\Rightarrow \log y - \log x = \log \eta$$

$$\Rightarrow \log y/x = \log \eta$$

$$\Rightarrow \eta = y/x .$$

Now,

$$p = \frac{\delta z}{\delta \eta} \cdot \frac{\delta \eta}{\delta x} + \frac{\delta z}{\delta \tau} \cdot \frac{\delta \tau}{\delta x} .$$

$$= \frac{\delta z}{\delta \eta} \left(-y/x^2 \right) + \frac{\delta z}{\delta \tau} \cdot 2x .$$

$$= \left(-y/x^2 \right) \frac{\delta z}{\delta \eta} + 2x \frac{\delta z}{\delta \tau} .$$